

When is it optimal to eradicate a weed invasion?

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Summary When a weed invasion is discovered a decision has to be made as to whether to attempt to eradicate it, contain it or do nothing. Ideally, these decisions should be based on a complete benefit-cost analysis, but this is often not possible. Partial analysis, combining knowledge of the demographics of the weed and economic techniques, can assist in making the best decision. This paper presents a general conceptual model to decide when eradication of a weed should be attempted. Decision rules are derived based on a few parameters that represent the rate of spread, the cost of controlling the invasion, and the cost of damage caused by the invasion. These decision rules are then used to identify the ‘switching point’ – the invasion size at which it is no longer optimal to attempt eradication. The decision rules are used to estimate the optimal duration of the eradication effort depending on the current size of the invasion. Sensitivity analysis is undertaken and the possibility of characterising an invasion based on five parameters is discussed.

Keywords Eradication, containment, economics, population models.

INTRODUCTION

When a weed invasion is discovered a decision has to be made as to whether to attempt to eradicate it immediately, contain it, or do nothing. The last two options must also be accompanied by a decision regarding how much additional information to gather on the potential spread of the weed and the possible damage caused by an uncontrolled invasion. This information can then be used to re-assess the initial decision. Ideally, these decisions should be based on a complete benefit-cost analysis, but this is often not possible.

Partial analysis, combining economic techniques, knowledge of the demographics of the weed and probabilities, can assist in making the right decision based on the information available.

This paper presents a conceptual model in which the benefits and costs from managing the spread of a weed population depend on the area invaded, the rate of spread and the current land use. The analysis is initially based on a static decision model, whereby radial spread is assumed and the eradication/containment decision is based on the size of the invasion. The concept of a ‘switching point’ (the point at which eradication is no longer an optimal option) is introduced.

The analysis is then extended by defining a dynamic decision model, where the optimal rate of control can vary between years and the planning horizon extends to infinity. The effects of cost and damage parameters on the location of the switching point are explored through sensitivity analysis. The paper concludes with a discussion of the possibility of characterising an invasion based on five parameters.

MATERIALS AND METHODS

Static decision model Sharov and Liebhold (1998) modelled the spread of a pest and its control using a barrier zone. They presented a general mathematical model, followed by a case study with gypsy moth spread in the USA. The first part of this paper is based on their model for the spread of small populations, those for which immediate eradication may be possible. The invasion is assumed to spread in a circular pattern and the size of the invasion is measured by its radius. The rate of spread can be slowed or reversed by targeting the invasion front. The decision as to whether to attempt to eradicate the invasion is based on evaluating net benefits (benefits – costs).

The cost of weed control is expressed in terms of the target rate of spread (Figure 1). The cost of slowing the spread (control cost) is zero when the invasion is allowed to spread uncontrolled (V_{max}). As the target rate of spread decreases (control intensity increases by moving left in Figure 1) the control cost increases. The cost function is expected to be convex to the origin

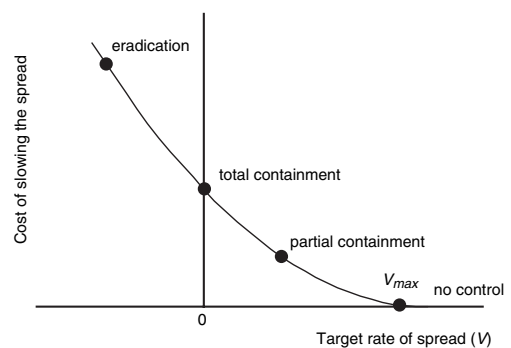


Figure 1. The cost of slowing an invasion that spreads at rate v_{max} in the absence of control.

because cheaper and easier control methods are used first, and more expensive options may be required for more intensive control. Where the cost curve crosses the vertical axis, spread is stopped (total containment is achieved). When the rate of spread is negative, the size of the invasion decreases with time and will eventually result in eradication.

The benefits of slowing the spread (Figure 2) are given by the difference in damages between the no-control alternative and the containment strategy i being evaluated.

Benefit of slowing the spread with strategy i
 = cost of damage experienced under strategy i
 – cost of damage in the absence of control.

A proper economic evaluation of the problem requires that benefits and costs be expressed in present-value terms. This in turn requires the analyst to decide on a planning horizon (T in years) and a discount rate (r percent). Net benefits in the static model were expressed in present value terms for a planning horizon of 25 years. The static decision model consisted of finding the rate of spread v^* that maximises net benefits for a given invasion size. This is a static decision model because the decision variable (v) is constant through time, even though the spread model is dynamic.

Dynamic decision model The static decision model has some desirable features that make it useful, the simplicity of finding an optimal solution being perhaps the most important. However, it may be advantageous to allow the intensity of control to change through time, at least during a transition period where the invasion is driven to a steady state. The steady state may represent eradication, containment or a stable weed population at carrying capacity.

The dynamic model developed here is an implementation of the ideas presented by Olson and Roy (2002). There are three key variables and three key functions that drive the invasion-control system. The three functions are the spread function, the control-cost function and the damage function. The variables are:

y_t = size of invasion (area);
 u_t = control (reduction in area invaded); and
 x_t = size of invasion after control is applied.

The optimal control strategy is determined by solving the following dynamic programming model.

$$V(y_t) = \min \left[\begin{matrix} C(u_t) + D(x_t) \\ + \delta \cdot V(y_{t+1}) \end{matrix} \right] \quad (1)$$

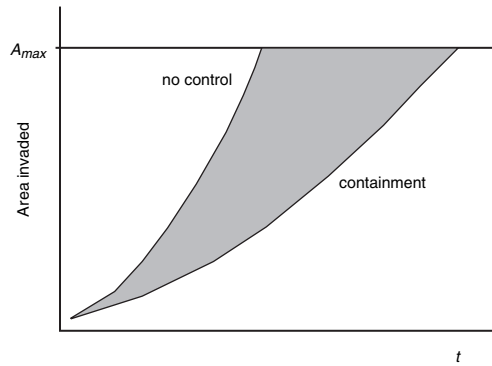


Figure 2. The benefits of slowing down an invasion are determined by the value of the shaded area between the no-control and containment lines.

subject to:

$$y_{t+1} = y_t + \Delta y_t(x_t) \quad (2)$$

$$x_t = y_t - u_t \quad (3)$$

This model is exactly equivalent to one maximising net benefits but it is more compact. V is the present value of the optimal policy; C and D are the cost and damage functions respectively; Δy_t is the spread function; and $\delta = 1/(1+r)$ is a discount factor based on the interest rate r . Note that control cost (C) is a function of the amount of control applied (u_t), whereas damage (D) is a function of the size of the invasion after control has been applied (x_t). Equation (1) represents the present value of the cost of the invasion, which includes current control costs + current damage + discounted future costs of the remaining invasion. Recursive solution of equations (1), (2) and (3) leads to an optimal decision rule that can be used to determine the conditions under which eradication is optimal.

Simple but plausible functional forms were used to represent the three key functions.

$$\Delta y_t = \alpha x_t \left[1 - \frac{x_t}{\kappa} \right] \quad (4)$$

$$C(u_t) = \beta_c u_t + \gamma_c u_t^2 \quad (5)$$

$$D(x_t) = \beta_d x_t + \gamma_d x_t^2 \quad (6)$$

Equation (4) is a logistic function and equations (5) and (6) are quadratic functions. Greek letters represent parameters to be estimated for the particular invasion. The working hypothesis is that all six

parameters are ≥ 0 and the spread parameters α and κ are > 0 . The implication of this hypothesis is that the invasion will eventually cover the entire area at risk if no control is applied. Definitions of the six parameters and their values are presented in Table 1. For simplicity it was assumed that the area at risk was 100 ha, so results can be interpreted as percentages.

RESULTS

Static model The static model was used to estimate the optimal strategy for any given invasion size, but the main output of this analysis is the switching point. The switching point is found at the intersection of two lines (Figure 3). The eradication line represents the net benefits from eliminating the invasion immediately after it is discovered. The containment line represents the net benefit of slowing down the invasion using the best control options available for the given invasion size.

At low invasion sizes eradication yields higher net benefits than containment, so it is optimal to eliminate the entire weed population. When the invasion radius is above a critical point (the switching point), it is optimal to contain rather than eradicate. The numbers in Figure 3 represent a hypothetical example and should not be taken as prescriptions for a general invasion. The exact location of the switching point will depend on the particular weed and the environment it invades; however the actual pattern shown in Figure 3 should have general applicability.

A problem with the static model is caused by the resilience and longevity of seed banks. It is unlikely that an invasion could be eliminated immediately after it is discovered unless it is very young. In the work of Sharov and Liebhold, immediate eradication makes sense because the invader was an insect. With weed invasions, eradication will generally be a long-term project. This raises three critical questions: (i) what is the optimal period over which the eradication effort should extend; (ii) what is the optimal intensity of control; and (iii) should the intensity of control change over time? These questions were answered by solving the dynamic model.

Dynamic model Solving the dynamic model results in an optimal decision rule (optimal control) which indicates the level of control (u_t) that should be applied for any given invasion size (y_t). Associated with the optimal control is the optimal state transition $y_t \rightarrow y_{t+1}$ which indicates whether the invasion increases ($y_t < y_{t+1}$), decreases ($y_t > y_{t+1}$) or remains stable ($y_t = y_{t+1}$) when subject to the optimal control. The optimal state transition can be used to determine the switching point in a dynamic context. This is shown below for a range of parameter values.

Table 1. Parameter definitions and values tested in this study.

Symbol	Values	Definition
κ	100	area at risk (ha)
α	0.1, 0.2	intrinsic rate of spread ($1\ y^{-1}$)
β_C	40–240	linear cost term ($\$ ha^{-1}$)
γ_C	0–25	quadratic cost term ($\$ ha^{-2}$)
β_D	5–20	linear damage term ($\$ ha^{-1}$)
γ_D	0–0.02	quadratic damage term ($\$ ha^{-2}$)

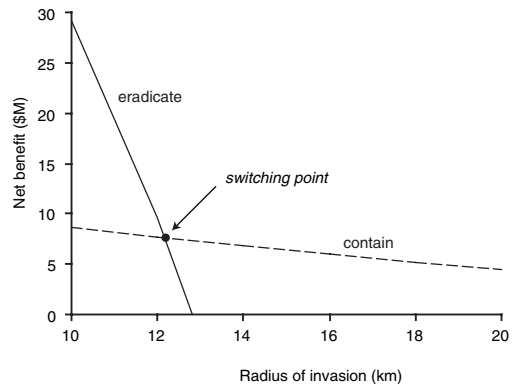


Figure 3. The switching-point is found at the intersection of the curves representing the benefits of eradication and the benefits of containment.

Case I – fast spread A value of $\alpha = 0.2$ implies a fast-spreading invasion, whereby the whole area at risk is invaded within 50 years. Under this assumption and with plausible cost and damage parameters the optimal state transition is presented in Figure 4. The optimal state transition should be interpreted by comparing it to a 45° ‘reference’ line, which represents the steady state (where $y_t = y_{t+1}$). This is a convenient way of exploring the dynamics of the optimization system and identifying equilibrium points. If the optimal state transition lies below the reference line the weed population is decreasing and, conversely, if it lies above the reference line the population is increasing. The switching point occurs where the reference line is intersected from below, which occurs at 55 ha in Figure 4. This means that if the invasion is discovered when its size is ≤ 55 ha (the switching point), it is optimal to eradicate it, otherwise it is optimal to allow it to spread.

The switching point occurs where the marginal cost of control exceeds the marginal cost of damage, with both costs measured in perpetuity and expressed in present-value terms.

The optimal state transition can be used to derive an optimal state path, which indicates the trajectory of the invasion through time under optimal control (Figure 5). With the assumed parameter values, an invasion under 55 ha should be eradicated within eight years, whereas an invasion over 55 ha should be allowed to run its course. Obviously, the switching point and optimal paths depend on a number of assumptions, including the six parameter values (Table 1) and the discount rate (r). So it is important to undertake sensitivity analysis.

Table 2 presents the switching points estimated for a range of cost and damage values when both equations (5) and (6) are linear (γ_C and $\gamma_D = 0$). For any control cost, increasing damage causes the switching point to increase from zero to 100. For example if the control cost is \$160 ha⁻¹ and damage is \$5 ha⁻¹, it is always optimal to do nothing, but if damage is \$7.50 ha⁻¹, it is optimal to eradicate invasions up to 57 ha in size.

When the control cost and damage functions are assumed to increase at an increasing rate (the functions are convex), a similar pattern arises as in the linear case (Table 3). The switching point increases as damage increases and it decreases as cost increases, but the changes are more pronounced than in the linear case.

Case II – slow spread A value of $\alpha = 0.1$ implies a slow-spreading invasion, whereby the whole area at risk is invaded within 100 years. Under this assumption the results are similar to those discussed above, except that any given level of damage requires lower control costs in order to make eradication the optimal decision (Table 4).

Certain combinations of parameter values produce interesting results where, in addition to the switching point, there is a ‘containment point’ (Figure 6). The containment point is a stable equilibrium where the optimal state transition intercepts the reference line from above. The resulting optimal state path (Figure 7) shows a similar pattern as the previous example, but with a higher switching point and the additional feature of a containment point. The presence of the containment point means that, even if the invasion is discovered when it occupies the whole area at risk (100 ha), it is optimal to decrease its size until it reaches the optimal containment point (about 85 ha).

DISCUSSION

An important question arising from the foregoing analysis is whether invasions can be characterised in terms of the five parameters α , β_C , γ_C , β_D and γ_D . This is arguably the simplest possible description of the decision problem. As seen above, α determines the

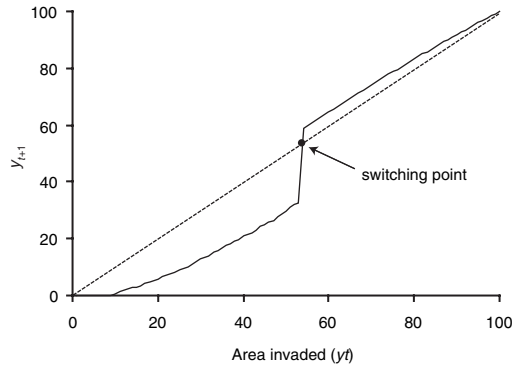


Figure 4. The optimal state transition (solid line) obtained by solving the DP model with $\alpha = 0.2$, $\beta_C = 200$, $\gamma_C = 2.0$, $\beta_D = 10$ and $\gamma_D = 0.01$.

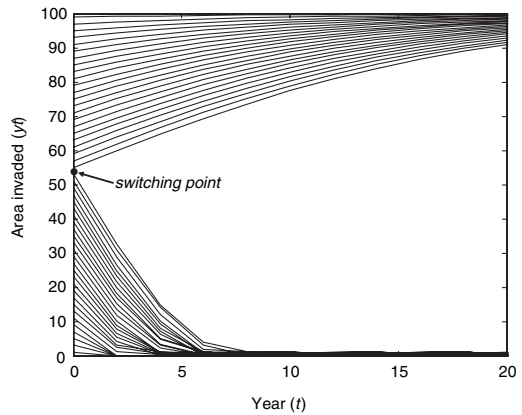


Figure 5. Optimal state paths for a range of starting values based on the rule in Figure 4.

Table 2. Switching points (ha)^a with cost and damage functions assumed linear ($\alpha = 0.2$, $\gamma_C = 0$, $\gamma_D = 0$, β_C and β_D vary).

Damage β_D (\$ ha ⁻¹)	Control cost, β_C (\$ ha ⁻¹)				
	160	180	200	220	240
5.0	0	0	0	0	0
7.5	57	44	0	0	0
10.0	83	71	61	54	44
15.0	100	100	100	92	83
20.0	100	100	100	100	100

^a a value of 0 means eradication is never optimal, 100 means eradication is always optimal, other values indicate the maximum invasion size that should be eradicated.

Table 3. Switching points (ha) with cost and damage functions assumed convex ($\alpha = 0.2$, other parameters vary as indicated).

Damage (β_D, γ_D)	Control cost (β_C, γ_C)				
	160, 1.6	180, 1.8	200, 2.0	220, 2.2	240, 2.4
$5, 5 \times 10^{-3}$	0	0	0	0	0
$7.5, 7.5 \times 10^{-3}$	51	42	0	0	0
$10, 10 \times 10^{-3}$	74	65	55	45	42
$15, 15 \times 10^{-3}$	100	100	96	84	74
$20, 20 \times 10^{-3}$	100	100	100	100	100

Table 4. Switching points (ha) with cost and damage functions assumed convex ($\alpha = 0.1$, other parameters vary as indicated).

Damage (β_D, γ_D)	Control cost (β_C, γ_C)				
	40, 14.4	45, 16.2	50, 18	55, 19.8	60, 21.6
$5, 5 \times 10^{-3}$	46	40	35	31	26
$7.5, 7.5 \times 10^{-3}$	72	63	57	51	46
$10, 10 \times 10^{-3}$	100	100	77	70	63
$15, 15 \times 10^{-3}$	100	100	100	100	100
$20, 20 \times 10^{-3}$	100	100	100	100	100

speed of spread and slower invasions are only worth eradicating if the damage is high or the cost of control is low. The shape of the cost function is determined by β_C and γ_C , whereas the shape of the damage function depends on β_D and γ_D . These four parameters require further discussion.

The most common way of calculating the cost of controlling an invasion is to multiply the cost per ha (e.g. cost of labour and chemicals) times the area to be treated. This implies that the cost function is linear and therefore $\beta_C > 0$ and $\gamma_C = 0$. This is a reasonable assumption for agricultural land, but may not be appropriate for inaccessible areas and natural ecosystems. In environments that cannot be treated in bulk (e.g. aerial spray) the search effort will be the critical factor for successful control. Search effort is likely to exhibit diminishing returns, which means that the last few weeds are more difficult to find than the first few weeds. This implies that $\gamma_C > 0$ for a system where search cost is important.

Damage is often calculated by multiplying the loss in the value of outputs caused by the weed times the total area invaded. This implies a linear damage function, with $\beta_D > 0$ and $\gamma_D = 0$. Again, this assumption is reasonable in the case of agriculture, where damage can be conveniently calculated as the difference in

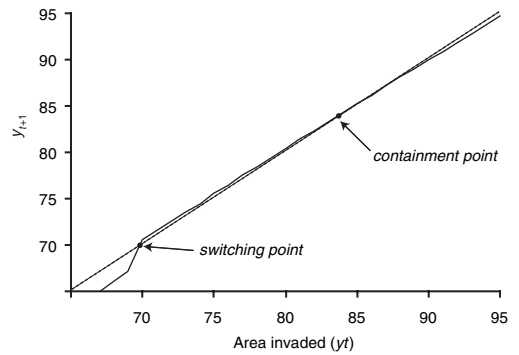


Figure 6. Optimal state transition with parameter values: $\alpha = 0.1$, $\beta_C = 50$, $\gamma_C = 20$, $\beta_D = 10$ and $\gamma_D = 0.01$.

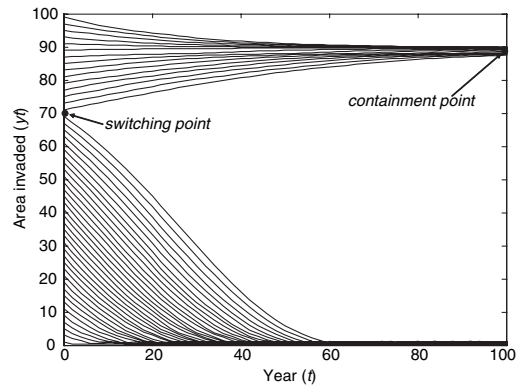


Figure 7. Optimal state paths for a range of starting invasion sizes based on the rule in Figure 6.

gross margins per ha before and after the weed invades. However, this may not be a realistic assumption for some natural environments, particularly those that have scarcity value. In these cases as the pristine area decreases the remaining uninvasioned area may have a higher value per unit area, so $\gamma_D > 0$.

Quadratic functions were used to represent costs and damages for convenience, as these curves can be linear or convex. It is possible that other functional forms with similar properties are more appropriate, but this is an empirical question, and calls for weed scientists to collect the data required to test the hypotheses advanced in this paper.

The control-cost function should be fairly easy to estimate, although the data are seldom readily available for existing eradication programs. The components of the cost function would be travel, labour, chemicals and equipment expenses. Estimating the damage function, however, is much more difficult.

Parker *et al.* (1999) identified impacts at five different levels:

1. individual effects, such as reduced agricultural yields and reduced growth of native plants;
2. genetic effects that occur when interbreeding changes the genetic makeup of the native population;
3. population and community effects; such as reduction in biodiversity;
4. ecosystems effects where a weed changes ecosystem processes; and
5. cumulative and indirect effects resulting from multiple invasive species.

Some of these effects (particularly 4 and 5) are clearly difficult to calculate. In theory, to estimate the damage function for any given weed invasion we would need to first assess which of these impacts are likely to apply and then assign a monetary value to each impact. This is not an easy task at the best of times and it is an unrealistic option when attempting to determine what to do about a new invasion before it spreads further.

Fortunately, performing sensitivity analysis with the dynamic model can answer questions such as: 'for the given cost of control, what would be the minimum damage that would make it optimal to eradicate the invasion?' In Tables 2–4 this question may be answered by selecting the control cost column and moving down to find the switching point corresponding to the current invasion size. This method would eliminate the need for full biodiversity valuations to make weed-control decisions in natural ecosystems. However, it would still be necessary to place a lower bound on the value of the ecosystem under threat.

Results are in line with the statement by Rejmanek and Pitcairn (2002) that 'attempts to eradicate widespread invasive species, especially those that do not have any obvious environmental impacts..., may not only be hopeless but also a waste of time and

resources'. This study provides some indication of where the decision will lie depending on estimated costs of damage and control.

A factor not considered here, but which is an important topic for further research, is the risk that the invasion will spread to other areas through long-distance transportation of seeds. The probability that the invasion will jump to sensitive areas is likely to affect the positions of the switching point and the containment point.

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